Correction (exam_juin_2012_3.pdf)

Ex2 :

1) $\varphi(x) = \varphi(0) + x \underbrace{\int_0^1 \varphi'(tx + (1-t)0)\,dt}_{\psi(x)}$

$\psi(x) = \int_0^1 \varphi'(tx)\,dt$

φ $\mathcal{C}^\infty$ $\Rightarrow$ ψ $\mathcal{C}^\infty$ et à support cpt

2) $-2 < \alpha < -1$

$\int_\varepsilon^{+\infty} x^\alpha \varphi(x)\,dx = \int_\varepsilon^{\cancel{A}+\infty} x^\alpha (\varphi(0) + x\psi(x))\,dx$ ~~avec supp φ ⊂ [-A,A]~~

$= \int_\varepsilon^{\cancel{A}+\infty} x^\alpha \varphi(0) + x^{\alpha+1}\psi(x)\,dx$

$= \int_\varepsilon^{\cancel{A}+\infty} x^\alpha \varphi(0)\,dx + \int_\varepsilon^{\cancel{A}+\infty} x^{\alpha+1}\psi(x)\,dx$

$= \frac{\varphi(0)}{\alpha+1}\left(\cancel{A^{\alpha+1}} - \varepsilon^{\alpha+1}\right) + \int_\varepsilon^{\cancel{A}+\infty} x^{\alpha+1}\psi(x)\,dx$

or ~~On~~ $= + A\,\varepsilon^{\alpha+1} + R_\varepsilon$

avec $A = -\frac{\varphi(0)}{\alpha+1}$ et $R_\varepsilon = \int_\varepsilon^{+\infty} x^{\alpha+1}\psi(x)\,dx$

or $\lim_{\varepsilon\to 0} \mathbb{1}_{]\varepsilon,+\infty[}(x)\, x^{\alpha+1}\psi(x) = \mathbb{1}_{]0,+\infty[}(x)\, x^{\alpha+1}\psi(x)$

$\left|\mathbb{1}_{]\varepsilon,+\infty[}(x)\, x^{\alpha+1}\psi(x)\right| \leq \mathbb{1}_{]0,+\infty[}\, x^{\alpha+1}\psi(x)\,dx \in L^1(\mathbb{R})$

car $-(\alpha+1) < 1$

TCD $\Rightarrow \lim_{\varepsilon\to 0} R_\varepsilon = \int_0^{+\infty} x^{\alpha+1}\psi(x)\,dx = \int_0^R x^{\alpha+1}\psi(x)\,dx$

3) a)

b) T est linéaire.

$|\langle T, \varphi\rangle| \leq \|\psi\|_\infty \int_0^R x^{\alpha+1}\,dx = \frac{R^{\alpha+2}}{\alpha+2} \leq \frac{R^{\alpha+2}}{\alpha+2}\|\varphi'\|_\infty$

$\Rightarrow$ T est $\mathcal{C}$ $\Rightarrow$ $T \in \mathcal{D}'(\mathbb{R})$.