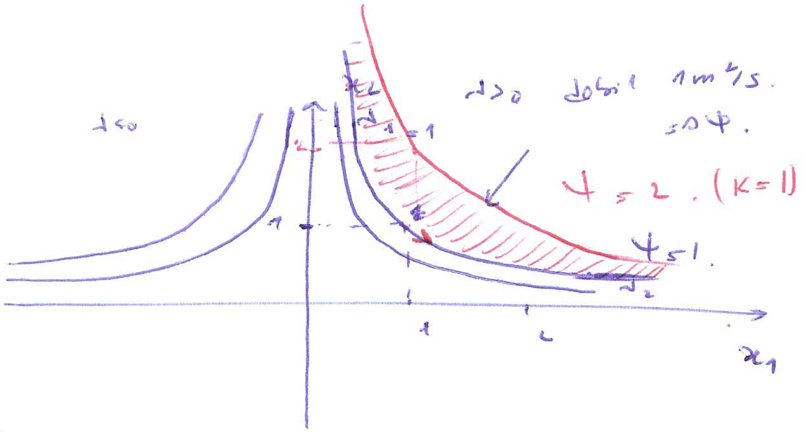
① MDF. Série 2 : 24/02/2014

Exercice 1



1/ $V_1 = K x_1$, $V_2 = -K x_2$

lignes de courant

$$\frac{dx_1}{U_1} = \frac{dx_2}{U_2} = \frac{dx_3}{U_3}$$

$$\frac{dx_1}{Kx_1} = \frac{dx_2}{-K x_2}$$

$$\int \frac{dx_1}{Kx_1} = -\int \frac{dx_2}{x_2} \Rightarrow \log|x_1| = -\log|x_2| + cste.$$

$$x_2 = \frac{d}{x_1}$$

2/

$$\begin{cases} V_1 = \dfrac{\partial\psi}{\partial x_2} = K x_1. & \Rightarrow \cancel{\psi = K x_1 x_2 + cste} \\ V_2 = -\dfrac{\partial\psi}{\partial x_1} = -K x_2. & \Rightarrow \cancel{\psi} = \end{cases}$$

$$\to \psi = K x_1 x_2 + c(x_1).$$

$$\frac{\partial\psi}{\partial x_1} = K x_2 = K x_2 + c'(x_1) \to c'(x_1) = 0 \Rightarrow c = cste.$$

$$\psi = K x_1 x_2 + c.$$

Si $\psi = 1$ lorsque $x_1 = x_2 = 1$. $\Rightarrow c = 1 - K$.

$$\to \boxed{\psi = K x_1 x_2 + (1-K)}$$

$$\psi = 2 \Rightarrow K x_1 x_2 + (1-K) = 2 \Rightarrow x_2 = \frac{1+K}{K} \cdot \frac{1}{x_1}$$

Exercice 2 :

$$\frac{dx}{u} = \frac{dy}{v} \Leftrightarrow \frac{dx}{u_0 \sin(w(t - \frac{y}{v_0}))} = \frac{dy}{v_0}$$

$$dx = \frac{u_0}{v_0} \sin(w(t - \frac{y}{v_0}))\, dy.$$

$$\Rightarrow x = \frac{u_0}{w} \cos(w(t - \frac{y}{v_0})) + cst.$$

2/ $t=0s$, $x=0$ et $y=0$

$0 = \frac{U_0}{\omega} \cos 0 + cte \rightarrow cte = -\frac{U_0}{\omega}$

$\Rightarrow x = \frac{U_0}{\omega} \left(\cos\left(\omega\left(t - \frac{y}{v_0}\right)\right) - 1 \right)$

$t = \frac{\pi}{2\omega}$ $\rightarrow$ $0 = \frac{U_0}{\omega} \cos\left(\frac{\pi}{2}\right) + cst = 0 \Rightarrow cst = 0$
$y=0$

$x = \frac{U_0}{\omega} \sin\left(\frac{\omega y}{U_0}\right)$

3/

$$\textcircled{0} \begin{cases} \frac{dx}{dt} = U_0 \sin\left(\omega\left(t - \frac{y}{v_0}\right)\right) \\ \frac{dy}{dt} = v_0 \quad \rightarrow y = v_0 t + cst \quad \Rightarrow \boxed{y = v_0 t + y_0} \end{cases} \quad (1)$$

$\frac{y}{v_0} = t + \frac{y_0}{v_0} \Rightarrow \frac{y_0}{v_0} = t - \frac{y}{v_0}$

$(1) \rightarrow \frac{dx}{dt} = U_0 \sin\left(\frac{\omega y_0}{v_0}\right) \Rightarrow \boxed{x = U_0 \sin\left(\frac{\omega y_0}{v_0}\right) t + x_0}$ (2)

4//.

$t=0$ (1) $\rightarrow y_0 = 0$ (1) + (2) $\rightarrow x = 0$ $\rightarrow \boxed{y = v_0 t}$

dans $t = \frac{\pi}{2\omega}$ (1) $\rightarrow$ $y_0 = \frac{-\pi v_0}{2\omega}$, $x_0 = \frac{v_0 \pi}{2\omega}$

5/.

$\hookrightarrow x = -U_0 t + \frac{U_0 \pi}{2\omega}$ et $y = v_0 t - \frac{\pi v_0}{2\omega}$

suite

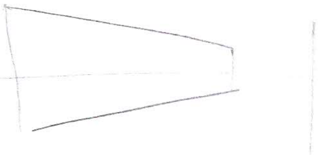
MDF

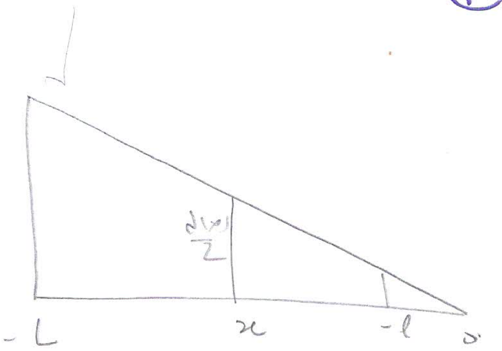
Série 2

03/03/2014

1

Exercice 4.





1/ $V(x)\, d(x) = V(-L) \cdot V_e$

$$\frac{-x}{L} = \frac{d(x)}{H_e} \Rightarrow d(x) = \frac{H_e\,(-x)}{L}$$

$$\vec{V}(x) = V(-L) \times \frac{H_e}{H_e} \cdot \frac{-L}{x}\, \vec{e_x} = V_e\left(-\frac{L}{x}\right) \vec{e_x}$$

2/ description eulérienne

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\,\vec{\nabla}.\vec{V}$$

$$a_i = \frac{\partial V_i}{\partial t} + V_j \frac{\partial V_i}{\partial x_j} \Rightarrow a_i = a_x = a = V(x)\, \underbrace{\frac{\partial V(x)}{\partial x}}$$

$$\frac{\partial V(x)}{\partial x} = V_e \frac{L}{x^2}$$

$$\Rightarrow \boxed{a = -\frac{L^2 V_e^2}{x^3}}$$

3/ $\frac{dx}{dt} = V = V_0\left(\frac{-L}{x}\right)$

$$\Rightarrow \int_0 x\,dx = -\int v_0 L\, dt$$

$$\frac{x^2}{2} = -(V_0 L)\,t + cte \quad \Big| \; CL \Rightarrow \frac{L^2}{2} = cte.$$

$$\Rightarrow x^2 = -2(V_0 L)\,t + \frac{L^2}{2} = L^2\left[1 - 2\left(\frac{V_0}{L}\right) t\right]$$

$$x = -L\sqrt{1 - 2\left(\frac{V_0}{L}\right) t} \Rightarrow x = f(X, t) \qquad -L = X_1$$

(X_1, t)

Lagrange

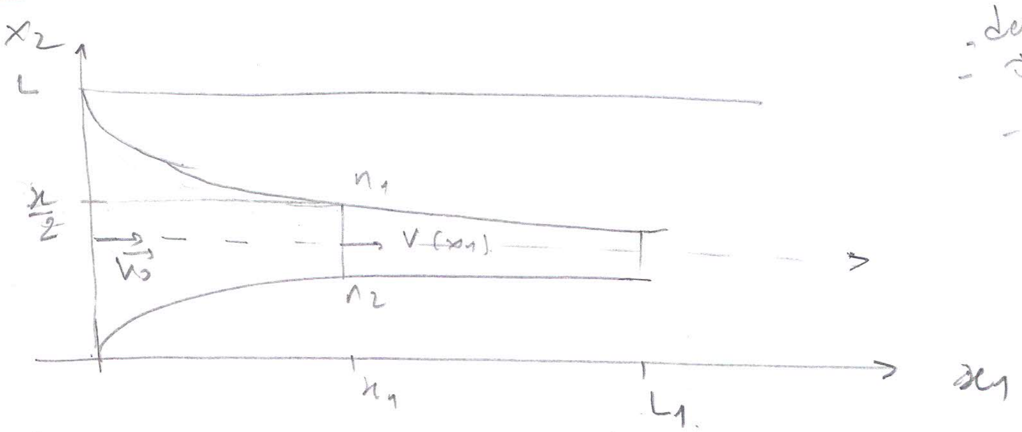
$$V(t) = \frac{dx}{dt} = V_0\left(1 - \frac{2(V_0)}{L}\, t\right)^{-1/2}$$

$$a(t) = \frac{dV}{dt} = \frac{V_0^2}{L}\left(1 - 2\frac{V_0}{L}\, t\right)^{-3/2}$$

$$= \frac{V_0^2}{L}\left(-\frac{x}{L}\right)^{-3} = -\frac{V_0^2 L^2}{x^3}$$

Exercice 4'

- des ct eule
- deriv particulaire
- cons de masse local / global



Conservation de masse

$\hookrightarrow V(x_1)\,\underbrace{l(x_1)}_{2(x_2)} = cte = L v_0 \rightarrow V(2x_1) = L v_0$

acc cste

$\hookrightarrow x_2 = f(x_1)$

$V \frac{dV}{dx_1} = a_0 \Rightarrow \int v\,dv = \int a\,dx_1$

$\boxed{V x_2 = \frac{L v_0}{2}}$

$a = \frac{dv}{dt} = cte$

$= V \underbrace{\frac{\partial v}{\partial x}}_{\frac{dv}{dx}} = cte = a_0$

$\frac{1}{2} V^2 = a_0 x_1 + cte$

$V = \left(\frac{L v_0}{2}\right) \frac{1}{x_2}$

$V\Big|_{x_2=0} = v_0 \rightarrow cte = \frac{1}{2} v_0^2$

$\frac{1}{2} V^2 = a_0 x_1 + \frac{1}{2} V_0^2 = \frac{1}{2} v_0^2 \left[1 + \frac{2a_0 x_1}{v_0^2}\right]$

$V = v_0 \sqrt{1 - \left(\frac{2a_0}{v_0^2}\right) x_1}$

$\frac{L V_0}{2 x_2} = v_0 \sqrt{1 - \frac{2a_0}{v_0^2} x_1} \Leftrightarrow \left(\frac{x_1}{L}\right) = \frac{1}{2} \frac{1}{\sqrt{1 + \left(\frac{2a_0}{v_0^2}\right) x_1}}$