Série 1 MDF 10/02/2014

①

Exercice 2 :

1/

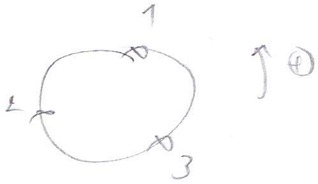
$$\varepsilon_{ijk} = \begin{cases} 1 & \text{si } (i,j,k) \text{ permutation paire de } (1,2,3) \\ -1 & \text{si } (i,j,k) \text{ —— impaire de } (1,2,3) \\ 0 & \text{sinon.} \end{cases}$$

$$\varepsilon_{ijk} \cdot \varepsilon_{pqk} = \varepsilon_{ij1}\,\varepsilon_{pq1} + \varepsilon_{ij2}\,\varepsilon_{pq2} + \varepsilon_{ij3}\,\varepsilon_{pq3}$$

$$i \neq j \neq 1 \rightarrow \begin{cases} i=2 \\ \text{et} \\ j=3 \end{cases} \text{ ou } \begin{cases} i=3 \\ \text{et} \\ j=2 \end{cases}$$

et

$$p \neq q \neq 1 \rightarrow \begin{cases} p=2 \\ \text{et} \\ q=3 \end{cases} \text{ ou } \begin{cases} p=3 \\ \text{et} \\ q=2 \end{cases}$$



Cas 1 : $(i=2$ et $j=3)$ et $(p=3$ et $q=2)$ valable.

Cas 2 : $(i=2$ et $j=3)$ et $(p=2$ et $q=3)$ ✓

$$\varepsilon_{ijk}\,\varepsilon_{pjk} \stackrel{?}{=} 2\delta_{ip}.$$

j indice muet $\overline{(1,2,3)}$

$$\varepsilon_{ijk}\,\varepsilon_{pjk} = \underbrace{\delta_{ip}\,\delta_{jj}}_{\delta_{ip}\ 3} - \underbrace{\delta_{ij}\,\delta_{jp}}_{\delta_{ip}} = 2\,\delta_{ip}$$

2/ $\vec{X} \times \vec{Y} = (\varepsilon_{ijk}\, X_j Y_k)\, \vec{e_i}$

$\vec{X} = X_i\ \vec{e_i}$

$\vec{Y} = Y_i\ \vec{e_i}$

$$\vec{X} \wedge \vec{Y} = \begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix} \wedge \begin{vmatrix} Y_1 \\ Y_2 \\ Y_3 \end{vmatrix} = \begin{vmatrix} X_2Y_3 - X_2Y_1. & = \varepsilon_{1jk}\, X_j Y_k \\ X_3Y_1 - X_1Y_3 & = \varepsilon_{2jk}\, X_j Y_k. \\ X_1Y_2 - X_2Y_1 & = \varepsilon_{3jk}\, X_j Y_k \end{vmatrix}$$

$\theta : (\vec{y}\ \vec{x})$

$\varepsilon_{ijk}\, x_j Y_k = \varepsilon_{i\,jk}\, Y_k\, X_j$

3) $\vec{C} \cdot (\vec{A} \times \vec{B}) \quad = \vec{C} \otimes \vec{B} \otimes \vec{A}$

? $\theta : [\vec{C} \otimes \vec{B} \otimes \vec{A}]$

$\vec{A} \wedge \vec{B} \Rightarrow \varepsilon_{ijk}\, A_j B_k$

$\vec{C} \cdot [\vec{A} \wedge \vec{B}] = \quad C_i [\varepsilon_{ijk}\, A_j B_k] \quad = \quad \varepsilon_{ijk}\, C_i A_j B_k.$

$= \vec{A} \otimes \vec{C} \otimes \vec{B} \qquad = \varepsilon_{ijk}\, B_k\, A_j\, C_i$

$= \vec{C} \otimes \vec{B} \otimes \vec{A}$

$\rightarrow \quad \theta : [\vec{B} \otimes \vec{A} \otimes \vec{C}]$

$\vec{\omega} \wedge \vec{x} = \varepsilon_{ijk}\, \omega_j x_k$

$= \varepsilon_{ijk} \left[\frac{1}{2} \varepsilon_{jpq} \Omega_{qp}\right] x_k$

$= \frac{1}{2} (\varepsilon_{ijk}\, \varepsilon_{jpq}) \Omega_{qp} X_k = \frac{1}{2} (\varepsilon_{kij}\, \varepsilon_{pqj}) \Omega_{qp} X_k.$

$= \frac{1}{2} (\delta_{kp} \delta_{iq} - \delta_{kq} \delta_{ip}) \Omega_{qp}\, x_k$

$= \frac{1}{2} [\delta_{kp} \delta_{iq} \Omega_{qp} X_k - \delta_{kq} \delta_{ip} \Omega_{qp} X_k)$

$= \frac{1}{2} [\Omega_{ik} X_k - \Omega_{ki} X_k] = \Omega_{ik}\, X_k$

cas particulier :

si $\Omega = \frac{1}{2} (\vec{\nabla} \vec{V} - (\vec{\nabla} \vec{V})^t)$

$\hookrightarrow \Omega_{ij} = \frac{1}{2} \left(\frac{\partial V_i}{\partial x_j} - \frac{\partial V_j}{\partial x_i} \right)$

$\omega_i = \frac{1}{2} (\vec{\nabla}_i \vec{V})_i \quad , \quad \vec{\omega} = \frac{1}{2} \underbrace{\mathrm{Rot}\ \vec{V}}_{\vec{\omega}\ :\ \text{vorticité}}$

Exercice 3 :

1 - $\vec{V} = \left(\frac{V_0}{H}\right) x_2 \, \vec{e_1}$

$$\vec{\nabla} \wedge \vec{V} = \begin{vmatrix} \partial/\partial x_1 \\ \partial/\partial x_2 \\ \partial/\partial x_3 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ \frac{V_0}{H}(x_2) \\ 0 \end{vmatrix} = -\frac{V_0}{H_3} \vec{e_3}$$

2/

$$\frac{\partial V_1}{\partial x_2} = \frac{V_0}{H}$$

$$\vec{\nabla}\vec{V} \longrightarrow \begin{pmatrix} 0 & \frac{V_0}{H} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

3/

$$D \Rightarrow \frac{1}{2}\left[\vec{\nabla}\vec{V} + (\vec{\nabla}\vec{V})^t\right] \Rightarrow \begin{pmatrix} 0 & \frac{V_0}{2H} & 0 \\ \frac{V_0}{2H} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$R \rightarrow \begin{pmatrix} 0 & \frac{V_0}{2H} & 0 \\ -\frac{V_0}{2H} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

4/ Appnumerique

5/

$$\frac{1}{r}\frac{dr}{dt} = \vec{k} \cdot D \cdot \vec{k}$$

$$N(\vec{e_1}) = (1\,0\,0) \begin{pmatrix} 0 & \frac{V_0}{2H} & 0 \\ \frac{V_0}{2H} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \delta t = 0 \Rightarrow$$ déf nulle selon $\vec{e_1}$

$$V(\vec{e_2}) = (0\,1\,0) \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0$$

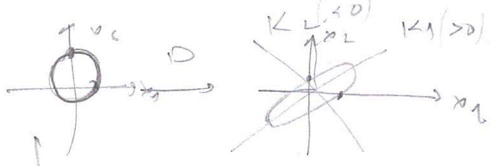
$$N\left(\vec{k_1} = \frac{1}{\sqrt{2}} e_1 + \frac{1}{\sqrt{2}} e_2\right) = \left(\frac{1}{\sqrt{2}} \ \frac{1}{\sqrt{2}} \ 0\right) \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$= \left(\frac{V_0}{2\sqrt{2}H_0} \ \frac{V_0}{2\sqrt{2}H_0} \ 0\right) \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$$= \left(\frac{V_0}{4H_0} + \frac{V_0}{4H_0}\right) \delta t = \frac{V_0}{2H_0} \simeq 500 \ m \cdot s^{-1}$$

$v(\vec{K}_2) = \dfrac{-V_0}{2H}$ AN: $-500\ m.s^{-1}$

6)



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échelle des particules.